Available online at www.sciencedirect.com
ScienceDirect
Physics Procedia 71 (2015) 389 – 393
Physics Procedia

18th Conference on Plasma-Surface Interactions, PSI 2015, 5-6 February 2015, Moscow, Russian Federation and the 1st Conference on Plasma and Laser Research and Technologies, PLRT 2015, 18-20 February 2015

Calculation of the parameters for a superconducting thin plate within Ginzburg-Landau theory

P. I. Bezotosnyi*, S. Yu. Gavrilkin, A. N. Lykov, A. Yu. Tsvetkov

Lebedev Physical Institute, Russian Academy of Sciences, Leninskii pr. 53, Moscow, 119991 Russia

Abstract

The behavior of a superconducting plate with transport current in a magnetic field parallel to its surface was studied by using numerical solution of Ginzburg-Landau (GL) equations. Boundary conditions for the order parameter in their general form have been used. The boundary conditions allow to consider the influence of the plate's boundaries on the superconducting state inside it. According to the calculations some features of the dependences of critical current and critical magnetic field in the parallel to the plate's surface direction as a function of the plate thickness have been detected. Such dependences are not explained by standard formulas for thin plates. On the basis of the calculations, an approach to estimate the coherence length ξ has been proposed. The results of the calculations are consistent with experimental data and qualitative analysis of the calculations within GL theory.

Peer-review under responsibility of the National Research Nuclear University MEPhI (Moscow Engineering Physics Institute)

Keywords: superconducting films; critical current; boundary conditions; Ginzburg–Landau theory; high temperature superconductors

* Corresponding author. Tel.: +7-499-132-67-67.
E-mail address: bezpi@sci.lebedev.ru

Peer-review under responsibility of the National Research Nuclear University MEPhI (Moscow Engineering Physics Institute)
doi:10.1016/j.phpro.2015.08.378

1. Introduction

The development of contactless methods for studying critical state of superconducting structures requires creation and development of the models which describe the processes taking place in such structures. One approach to the description of the superconducting state is based on Ginzburg-Landau (GL) theory. The advantage of this approach, in particular, includes an ability to describe magnetic and transport properties of superconductors, and to describe vortex state in a superconducting sample. Unfortunately, the equations of the theory are quite complicated and allow to get the solution by analytical methods in a very limited number of cases. Numerical methods allow to extend the considered range of cases.

Within our work, the state of the superconducting plate with transport current in the parallel to its surface magnetic field was studied using a numerical solution of the GL equations. The method used for determination of the self-consistent solution of the system of GL equations allows to obtain parameters applied to the entire plate (for example, the critical current on the external magnetic field or the temperature dependences of the critical magnetic field and critical current density) and distribution of the parameters for the plate thickness.

The approach to the description of real superconducting structures within GL theory was developed by taking into account the influence of boundaries on superconducting properties of the material for structures of finite size, such as a thin plate Ref. Bezotosnyi et al. (2014), Bezotosnyi et al. (2015). The boundary influence is important for high-temperature superconductors, which have a layered crystal structure Ref. Andryushin et al. (1993). The boundary conditions for the order parameter were formulated by Andryushin et al. (1993) and were obtained from the free energy minimum principle taking into account sample surface influence. Such boundary conditions for the order parameter were used in our work.

2. Problem formulation

In our study, the system of the GL equations was solved by numerical methods. The case of a long and wide superconducting plate of thickness D in magnetic field H was considered. The Cartesian coordinate system (x, y, z) with y and z axes parallel to the plate surface plane and the z axis is parallel to the external magnetic field was used. The transport current flows along the y axis. As a transport current I_t the current per plate unit width was used. Using the ordinary method for choosing the vector potential **A** calibration, we can write the GL equations in the form

$$\frac{d^2\psi}{d{x_\xi}^2}+(\psi-\psi^3)-\frac{U^2}{\kappa^2}\psi=0 \quad (1)$$

$$\frac{d^2U}{d{x_\xi}^2}-\frac{\psi^2}{\kappa^2}U=0 \quad (2),$$

$\kappa=\lambda/\xi$ is the Ginzburg–Landau parameter, λ is the magnetic field penetration depth, ξ is the coherence length, $x_\xi = x/\xi$, and ψ is the normalized order parameter,

$$\psi=\frac{\Psi}{\Psi_0},$$

Ψ_0 is the order parameter in the bulk superconductor at zero external magnetic field. In considered geometrical formulation, the vector potential has only the y-component, $\mathbf{A} = \mathbf{e}_y A(x)$. Instead of dimensional values of the potential A, field induction B, and current density j_S in the superconductor, the dimensionless quantities $U(x_\xi)$, $b(x_\xi)$, and $j(x_\xi)$ are introduced

$$A=\frac{\phi_0}{2\pi\kappa\xi}U,\quad B=\frac{\phi_0}{2\pi\kappa^2\xi^2}b,\quad j_S=\frac{c\phi_0}{8\pi^2\kappa^3\xi^3}j \quad (3),$$

φ_0 is the flux quantum.

For the order parameter equation (1), we choose the boundary conditions in the form

$$\frac{d\psi}{dx_\xi}\Big|_{x_\xi=0}=\frac{\psi(0)}{\Lambda},\frac{d\psi}{dx_\xi}\Big|_{x_\xi=d}=-\frac{\psi(d)}{\Lambda} \qquad (4),$$

$d = D/\xi$, Λ – parameter of the plate surface. This parameter was considered in scale of the coherence length ξ.

Since the transport current I_t in the plate induces a magnetic field:

$$H_I=\frac{2\pi}{c}I_t,$$

the total field near the plate surfaces is $H \pm H_I$, and the boundary conditions to Eq. (2) are written as

$$\frac{dU}{dx_\xi}\Big|_{x_\xi=0}=b\,\Big|_{x_\xi=0}=h-h_I,\frac{dU}{dx_\xi}\Big|_{x_\xi=d}=b\,\Big|_{x_\xi=d}=h+h_I,$$

where

$$h=\frac{H}{H_\xi},h_I=\frac{H_I}{H_\xi},H_\xi=\frac{\phi_0}{2\pi\kappa^2\xi^2}.$$

The magnetic field penetration depth λ and coherence length ξ depend on temperature. Therefore, the presented dependences are implicit functions of temperature and are formally valid at any temperature T. However, the GL equations themselves are applicable only in the limit $T \to T_c$.

The current density J was considered as the ratio of the transport current value to the plate thickness d. The value of the critical current density J_c and critical field h_c for the superconducting plate was assumed to be equal to the value of the current density J or an external field h in which the order parameter becomes zero $\psi(x_\xi) = 0$ everywhere in the plate.

Initially, an approach was used to consider Meissner state the plates as a one-dimensional problem. However, there is a transition from one-dimensional to two-dimensional problem, describing vortex state of the plates, resulting in the appearance of the second symmetric solutions for the one-dimensional order parameter Ref. Bezotosnyi et al. (2014). As a result, it is possible to obtain the distribution of the order parameter for vortices in a superconductor.

3. Results of the numerical calculation

The dependences of the critical current density and parallel critical magnetic field on the plate thickness were calculated. General boundary conditions with finite Λ and $\Lambda=\infty$ were used in numerical calculations of the GL equations. The dependences of the critical current density on the plate thickness for the different values of the parameter of the plate surface Λ are shown in Fig. 1 (a). The calculation was made for the case κ =10. The Ginzburg-Landau parameter κ for most superconducting materials used today in practice has the level of 10 or more, so the case κ =10 is interesting from the point of proximity to the practically used materials. As the figure shows, the decrease in the critical current density with decreasing plate thickness is observed in the case. This effect was repeatedly observed in experimental works; however, as a rule, it was assumed that the decrease in the critical current density with decreasing plate thickness is caused by plate structure degradation: ultrathin films (plates) cease to be homogeneous and become island ones. Generally, it is believed that the critical current density does not change with decreasing plate's thickness. However, the results of numerical solution of the system of the GL equations with boundary conditions of the general type with finite Λ find similar with practical data dependence of the critical current density on the thickness of the plate, and for the case $\Lambda=\infty$ gets by well-known result of the constancy of the density of the critical current with changing plate's thickness.

The dependences of the parallel critical magnetic field as a function of the plate thickness are shown in Fig. 1 (b). Simple analytical calculations in assumption of the order parameter constancy on the thickness of the plate show that parallel critical field increases with decreasing thickness according to the law $1/d$. Indeed, our calculations for the case $\Lambda=\infty$ demonstrate this result. In fact, the dependences of the critical magnetic field on the thickness weakly changed at values of Λ over 200. However, at finite values of Λ there is a maximum of the critical magnetic field at a certain thickness. The value of the critical field begins to sharply decrease with further decrease in the thickness.

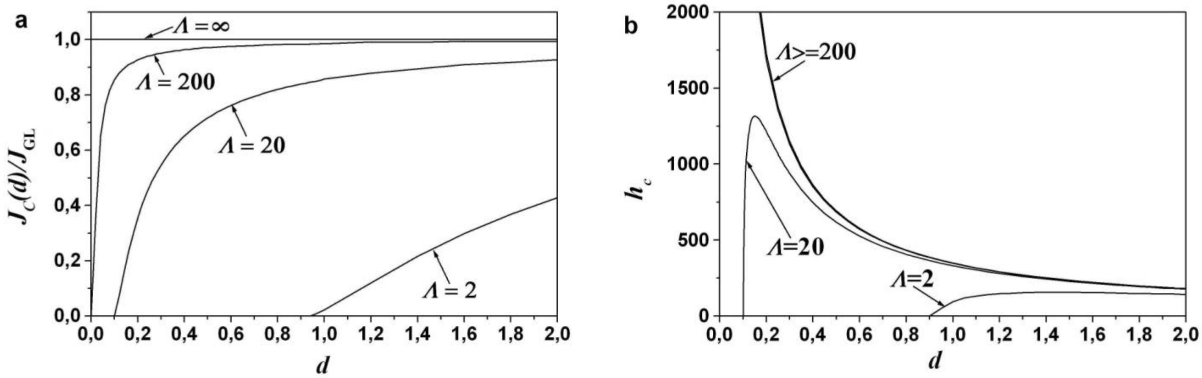

Fig. 1. (a) The dependence of the critical current density J_c (ratio of the critical current to the plate thickness d), normalized by the current density of Ginzburg-Landau depairing current J_{GL}, on the thickness of the plate d for different values of the boundary parameter Λ. In this case, the external magnetic field $h = 0$, $\kappa = 10$. (b) The dependence of the parallel to the surface of the plate critical field h_c on the thickness of the plate d for different boundary parameters Λ. In this case, the transport current is absent, $\kappa = 10$.

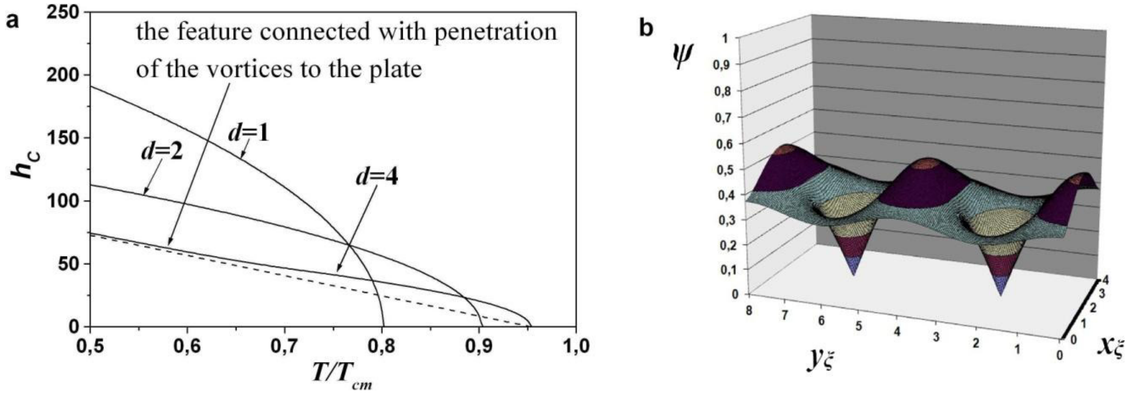

Fig. 2. (a) The dependence of the parallel critical field h_c as a function of the ratio T/T_{cm} for superconducting plates with different thickness d. Here T_{cm} - critical temperature of the bulk sample, $\kappa = 10$ and $\Lambda = 10$. The point at which the solid lines begin to diverge with dashed lines for plates with d=4 - point associated with the nucleation of vortices in the sample; (b) The distribution of the modulus of the order parameter ψ in the plate in the vortices case. The x axis is directed into the depth of the plate, the y axis is directed along the surface of the plate. Here d=4, κ=10, Λ =10. The coordinates are given in units of ξ. The external magnetic field is $80H_\xi$.

Analysis of the dependences of the critical field and the critical current density on the thickness shows that there is some critical plate thickness at which critical current vanishes, and thus, superconductivity not exist in this plate.

This value depends on Λ (compare Fig. 1 (a) and 1 (b)). The formula for this thickness was previously obtained analytically by Andryushin et al. (1993). The critical thicknesses calculated by the analytical formula are the same as those that are visible in the dependences obtained by numerical calculations.

Fig. 2 (a) shows the temperature dependences of the parallel critical magnetic field for different values of the plate thicknesses d. The dependence for the case d=4 represents changing type of the dependence from root character to linear. The root dependence is typical for Meissner state of the plate. The linear dependence is typical for vortices state of the plate (shows by dash in figure). The nucleation of the vortices in the plate is proved by the order parameter calculations (Fig. 2 (b)). The existence of such a feature can be used to estimate the coherence length ξ of the plate. The ratio of plate thickness to the coherence length has a certain value for every parameter of the plate surface Λ. Such features were previously observed in the experiment Ref. Shabanova et al. (2007).

4. Conclusions

The main results of the study can be formulated as follows.

Numerical methods were used for solving the system of the GL equations with the boundary conditions in general form for the order parameter. Such type of the boundary conditions allows to consider the influence of the plate's boundaries on the superconducting state inside it. The influence of the boundaries on the superconducting state inside plate corresponds to the case of finite Λ.

The dependences of critical current and critical magnetic field of superconducting plates as a function of its thickness have been obtained. These dependences for the case of finite Λ are different from well-known classical dependences and show existence of the critical plate thickness at which critical current vanishes, and thus, superconductivity does not exist in these plates.

The temperature dependences of the parallel to the surface of the plate critical magnetic field for different values of the plate thicknesses were obtained. The features connected with the nucleation of the vortices in the plate on these dependences are observed for particular thicknesses of the plate. The conclusions are supported by the order parameter calculations. The features can be used to estimate the coherence length ξ of the plate for different values of the parameter of the plate surface Λ.

Acknowledgements

This work is supported by Russian Foundation for Basic Research (grant № 14-08-31278).

References

Andryushin, E. A., Ginzburg, V. L., Silin, A. P., 1993. Physics-Uspekhi (Advances in Physical Sciences) 36, 854–857.
Bezotosnyi, P. I., Gavrilkin, S. Yu., Lykov, A. N., Tsvetkov, A. Yu., 2014. Bulletin of the Lebedev Physics Institute No. 6, 153-159.
Bezotosnyi, P. I., Gavrilkin, S. Yu., Lykov, A. N., Tsvetkov, A. Yu., 2014. Bulletin of the Lebedev Physics Institute No. 1, 1-6.
Shabanova, N. P., Krasnosvobodtsev, S. I., Varlashkin, A. V., Golovashkin, A. I., 2007. Physics of the Solid State 49 (2007) 1040-1045.